Finite Element Modeling of deformable NEMS taking into account mechanical contact

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***Abstract*— This work focuses on the modeling of mechanical contact applied to Nano-Electro-Mechanical-Systems (NEMS). A magneto-elastic formulation associated to the definition of unilateral contact in static case is presented. This model is discretised by finite element method and a numerical study of a magnetic nano switch is performed. FEM results are compared to those of a semi-analytical model developed for optimization processes.**

I. INTRODUCTION

Magnetic NEMS based instruments and devices are at the forefront of nanotechnology, enabling fundamental measurements and standards and new types of devices related to magnetism. This is given by low voltage and power consumption with large actuation distance which provides a number of advantages compared to electrostatic NEMS [1]. In NEMS technology, magnetic nano switches have many applications as nanomechanical memory, power switches,... Their working principle is based on the deflection of a beam submitted to the influence of a magnetic field. The mechanical contact between beam and substrate is an important parameter for the behavior of these nano switches [2]. Indeed, contact quality strongly depends on roughness of contact area and on surface and contact forces. Nevertheless, their modelings are often strongly approximated [3]. We propose in this paper a magneto-elastic model included unilateral contact formulation and its discretisation by the finite element method. The modelisation is applied to a magnetic nano switch, and the numerical results are compared with a semi-analytical model realizing the magnetic structural coupling considering contact analysis.

II. FINITE ELEMENT MODELING

The problem under consideration involves the contact of an elastic body, submitted to magnetic forces, with a rigid (or elastic) body in static case (Fig. 1).

A. Contact problem description

Consider two elastic bodies occupying a bounded domain, Ω_1 and Ω_2 (Fig. 1), of $\mathcal{R}^d$ with $d = 2$ or 3. The bodies are subjected to body forces $\boldsymbol{f}$ and to prescribed tractions $\boldsymbol{t}$ and displacements $\boldsymbol{u}_0$ on the part Γ_σ and Γ_u of the boundary Γ respectively. For the two bodies, the potential contact boundaries are Γ_{c_1} and Γ_{c_2}. For a classical linear elasticity problem, the equations are:

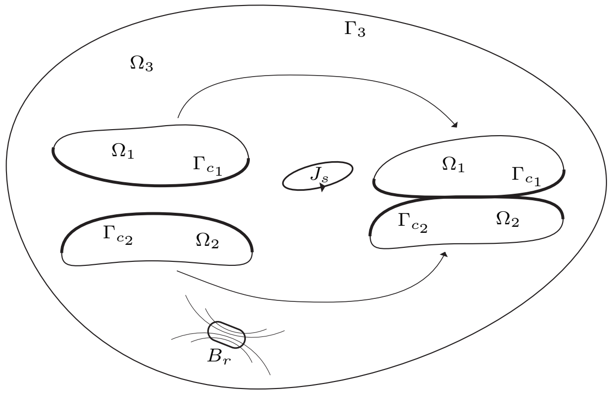

Fig. 1. Study domain of magneto-mechanical contact problem ($\Omega = \Omega_m \cup \Omega_3$ and $\Omega_m = \Omega_1 \cup \Omega_2$)

$$\begin{cases} \nabla \cdot \boldsymbol{\sigma} + \boldsymbol{f} = 0 & \text{in } \Omega_m \\ \boldsymbol{\sigma} = \boldsymbol{C} : \boldsymbol{s} & \text{in } \Omega_m \\ \boldsymbol{\sigma}(\boldsymbol{u}) \cdot \boldsymbol{n} = \boldsymbol{t} & \text{on } \Gamma_\sigma \\ \boldsymbol{u} = \boldsymbol{u}_0 & \text{on } \Gamma_u \end{cases} \quad (1)$$

where $\boldsymbol{n}$ is the outward unit normal vector, $\boldsymbol{C}$ the stiffness tensor and $\boldsymbol{s}$ the strain tensor defined in the assumption of small displacement by:

$$\boldsymbol{s}(\boldsymbol{u}) = \frac{1}{2}\left(\nabla \boldsymbol{u} + \nabla \boldsymbol{u}^t\right) \quad (2)$$

To state the laws of contact and friction, the displacement and tractions vectors are decomposed on Γ_c into normal and tangential components:

$$\begin{aligned} \sigma_n &= \sigma_{ij}\, n_i\, n_j & \sigma_{t_i} &= \sigma_{ij}\, n_j - \sigma_n\, n_i \\ u_n &= u_i\, n_i & u_{t_i} &= u_i - u_n\, n_i \end{aligned} \quad (3)$$

For a frictionless contact, the unilateral contact conditions on the boundaries Γ_c, which implies that no boundary point of the first body may penetrate the other, are defined by :

$$\begin{cases} g_n = (\boldsymbol{u}_2 - \boldsymbol{u}_1) \cdot \boldsymbol{n} \geq 0 \\ \sigma_n = \sigma_{n_1} = -\sigma_{n_2} \leq 0 \\ \sigma_n \cdot g_n = 0 \end{cases} \quad (4)$$

with g_n the signed normal distance. The first condition states that no penetration may occur. Hence, this is the form in which the impenetrability constraint is cast. Using this, the normal traction can be characterised. The second condition states that the contact normal traction should be compressive. Finally, the third condition states a complementarity condition. If there is no contact, then no compressive tractions can occur. Alternatively, if there are no compressive stresses, then the distance must be positive.

B. Magneto-elastic contact problem formulation

In the static case, the formulation of the magneto-elastic contact problem, without magnetostrictive phenomena [4], can be established from a minimisation of the functional energy $\mathcal{F}$ in terms of magnetic flux density $\boldsymbol{b}$ and strain $\boldsymbol{s}$:

$$\mathcal{F}(\boldsymbol{b}, \boldsymbol{s}) = W(\boldsymbol{b}, \boldsymbol{s}) - T \quad (5)$$

where $W(\boldsymbol{b}, \boldsymbol{s})$ and T are respectively the magneto-elastic energy and the work of magnetic and mechanical sources, defined by:

$$W(\boldsymbol{b}, \boldsymbol{s}) = \int_{\Omega} \left(\int_0^b \boldsymbol{h}(\boldsymbol{b}')d\boldsymbol{b}' + \int_0^s \boldsymbol{\sigma}(\boldsymbol{s}')d\boldsymbol{s}' \right) d\Omega \quad (6)$$

$$\begin{aligned} T = & \int_{\Omega} \boldsymbol{a} \cdot \boldsymbol{j} \, d\Omega + \int_{\Gamma_h} \boldsymbol{a} \cdot (\boldsymbol{h} \times \boldsymbol{n}) \, d\Gamma_h \\ & + \int_{\Omega_m} \boldsymbol{u} \cdot \boldsymbol{f} \, d\Omega + \int_{\Gamma_\sigma} \boldsymbol{u} \cdot (\boldsymbol{\sigma} \cdot \boldsymbol{n}) \, d\Gamma_\sigma \end{aligned} \quad (7)$$

with $\boldsymbol{a}$ the magnetic vector potential and $\boldsymbol{j}$ the current density. Application of variational principles, and taking into account the contact conditions (4), gives the following magnetic and mechanical formulations associated to arbitrary variations $\boldsymbol{\delta a}$ and $\boldsymbol{\delta u}$:

$$\int_{\Omega} \nabla \times \boldsymbol{\delta_a} \, \boldsymbol{\nu} \, \nabla \times \boldsymbol{a} \, d\Omega + \int_{\Gamma_h} \boldsymbol{\delta_a} \cdot (\boldsymbol{h} \times \boldsymbol{n}) \, d\Gamma = \int_{\Omega} \boldsymbol{\delta_a} \cdot \boldsymbol{j} \, d\Omega \quad (8)$$

$$\int_{\Omega_m} s(\boldsymbol{\delta u}) \boldsymbol{C} \boldsymbol{s} - \boldsymbol{\delta u} \cdot \boldsymbol{f} d\Omega - \int_{\Gamma_\sigma} \boldsymbol{\delta u} \cdot \boldsymbol{t} d\Gamma - \boldsymbol{\delta u} \left(W(\boldsymbol{b}, \boldsymbol{0}) \right) \geq 0 \quad (9)$$

The inequality (9) is due to the contact definition and define the variational inequality of the magneto-elastic contact problem. The last term of the right-hand side of (9) are the nodal magnetic forces determined by the local derivative of magnetic energy [5]. Resolution are realizing by a penalty method, the Lagrange multiplier method or an augmented lagrangian method [7].

III. Semi-Analytical Modeling

Thanks to basic geometries of NEMS, magnetic fields radiated by permanent magnets and conductors are computed by using pure algebraic equations through Coulombian equivalent surface charge approach and Biot-Savart law. Surface and volume numerical integrations are used to compute magnetic forces and torques.

A semi-analytical structure model compute the deflection, due to magnetic forces and torques, of the cantilever beam in the presence of contact. It is based on assumptions of small displacement, which allows reducing the cantilever beam 3D to 1D problem, on linear and isotropic materials and on considering only one direction of deformation. To compute the deformation, the 1D-beam is split into a set of segments, and deformation of each segments is calculated by the following equation:

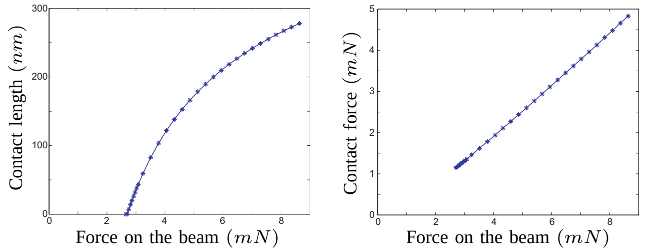

Fig. 2. Results of the semi-analytical structural contact model

$$E\,I \frac{\partial^2 s}{\partial x^2} = M \quad (10)$$

where E is the Young's modulus, I the second moment of area and M the bending moment. Adequate boundary conditions and connection conditions are used to ensure continuous deformation. In application of the superposition principle, the total deformation is the sum of the deformations of each segments created by each forces and torques.

Model of contact is inherently coupled with the structural model by replacing the contact by a distribution of force on contact. The contact force F_c is computed by:

$$F_c = F_t - F_r \quad (11)$$

where F_t is the total force applied on the beam and F_r the reaction force on fixed side of the beam. As the distribution of forces, which replaces the contact, is unknown, an iterative method is used. Figure 2 shows the variation of the contact length and contact force as function of forces applied on the beam.

IV. Conclusion

In this paper, the formulation of a magneto-mechanical contact problem for a finite element analysis has been presented. In the full paper, this model will be developed and particularly the finite element discretisation, the numerical resolution thanks to regularization methods and the procedure to improve the contact management. Comparisons with the semi-analytical model will be made for a magnetic nano switch.

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